force distribution will be modified by the vortex pattern, and the modification will be a function of the fin location in the vortex pattern. If the forces on fins b and d are different, then a resultant lift (side) force exists. In general, it has been found that the resultant side force acts opposite to the body magnus force. (The forementioned reasoning has assumed a roll position where fins b and d are in the angle-of-attack plane. Similar reasoning using all four fins can be used at other roll positions.)

If η_d and η_b are considered to be due only to the influence of the wing-vortex pattern, then an additional term must be added to Eqs. (27) and (29) of Ref. 1 to account for the lift (side) forces due to rotation of the fins. Also, when a fin is immersed completely in the vortex pattern, its lift need not be equal to the wing lift as stipulated in Eq. (32) of Ref. 1 $[\eta_b(\alpha_d) = 1]$. Its lift will be a function of the fin position in the vortex pattern. Since the details of the wing-body vortex or wake pattern have not been computed, it appears that the fin forces cannot be computed easily.

During the past year a series of wind tunnel tests on a finned spinning missile have been run in the Ballistic Research Laboratories' tunnels with the objective of obtaining magnus forces and moments. This configuration does not have wings, and the fins are the same diameter as the major body diameter. The configuration depends on the body-fin interference to produce the magnus forces and moments. Sufficient room is not available here to describe these tests, but the results have been published in Ref. 2.

References

[1] Benton, E. R., "Wing-tail interference as a cause of 'magnus' effects on a finned missile," J. Aerospace Sci. 29, 1358–1367 (1962).

[2] Platou, A. S., "The magnus force on a finned body," Ballistic Research Lab. Rept. 1193 (March 1963).

Reply by Author to A. S. Platou

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The author of Ref. 1 wishes to thank Platou for his instructive comments.[2] However, they do not basically disagree with the original paper but rather lead to some higher-order corrections to the theory. The main implication of Ref. 2 is that the effects of roll rate were not properly accounted for in Ref. 1 when the forces and moments on rotating wing and tail panels were dealt with. Actually, these effects were considered, but they were found to be much smaller than the main effects described.

In his second paragraph, Platou correctly points out that the rotation of a wing panel produces a distribution of effective angle of attack that varies linearly with spanwise distance from the axis of rotation. The loading that accompanies this distribution is also proportional to roll rate, and it must be added to that arising from wing stationary factors. Consequently, there is a force and a moment "due to roll rate." This was discussed in Sec. 5 of Ref. 1 (see pp. 1362–1363). The moment, of course, is just the well-known roll damping moment. When it is added to the rolling moment due to wing deflection, the total rolling moment on each panel is found to be zero, as it must be under free spin conditions (refer to last paragraph on p. 1362, Ref. 1).

The real point raised by Platou is that, even though the rolling moment is zero, there is a net lift force on each wing panel. This is correct, but, as implied in Ref. 1 (top of p. 1363), this net force is, to a first approximation, negligible. In fact, it can be shown to be considerably smaller than either of the component forces (due to panel deflection and roll rate), which make up the net panel force. References 3 and 4 imply that the spanwise centers of pressure for the forces due to deflection and roll rate are at about 0.76 and 0.60 of the exposed semispan, respectively. Together with the fact that the net moment on each panel is zero, this implies that, for the present missile, the net lift force is less than $\frac{1}{5}$ of the force due to panel deflection. Moreover, the net forces on panels 2 and 4 are directed oppositely, so the total side force is indeed small. This net side force is zero at $\alpha = 0$, by symmetry. Furthermore, Fig. 4 of Ref. 1 shows that experimentally, it remained zero for all α up to 8°. This further confirms the belief that these net side forces on each panel are considerably smaller than the primary forces dealt with. Whereas such forces are unimportant for the wing-body combination, a more accurate theory than that of Ref. 1 would attempt to take into account the influence of these forces in determining tail loads.

The rest of Platou's comment is quite correct, but it applies to a range of interest deliberately excluded from Ref. 1. The tail interference factors used in Eqs. (27) and (29) of Ref. 1 do not include the effects of roll rate. This should involve no appreciable error so long as all four tail panels mostly are immersed in the vortex wake. This condition holds for the range of α between 0 and 8°, which is the only range discussed in Ref. 1. For higher α, the forces and moments become nonlinear with angle of attack. Certainly, one of the nonlinear effects to be expected is the one surmised by Platou, which occurs when one panel experiences free-stream conditions while the other one still is immersed in the wake.

Finally, it should be pointed out that the condition expressed in Eq. (32) of Ref. 1 is not a stipulation, as stated in Ref. 2, but rather an assumption of the theory which is plausible (at least to the author), and whose ultimate justification is that it predicts results of the right magnitude. Actually, as mentioned in the paragraph immediately following Eq. (32), it is not assumed that the lift of one tail panel is the same as on one wing panel but only that the total side load developed on the two relevant tail panels is equal in magnitude to the lift on one wing panel.

References

[1] Benton, E. R., "Wing-tail interference as a cause of 'magnus' effects on a finned missile," J. Aerospace Sci. 29, 1358–1367 (1962).

[2] Platou, A. S., "Comments on 'Wing-tail interference as a cause of 'magnus' effects on a finned missile,' " AIAA J. 1, 1963–1964 (1963).

[3] Rogers, A. W., "Application of two-dimensional vortex theory to the prediction of flow fields behind wings of wing-body combinations at subsonic and supersonic speeds," NACA TN 3227 (September 1954).

[4] Bird, J. D., "Some theoretical low-speed span loading characteristics of swept wings in roll and sideslip," NACA Rept. 969 (1950).

Motion in a Soap Film

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Nomenclature

h = thickness
σ = specific weight
T = surface tension
r,φ,x = coordinates

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v = velocity of flow
t = time
g = gravitational constant
μ = kinematic viscosity

IN a previous paper,[1] Galletly asks for comments on his soap film paradox. The paradox arises when the forces on a part of the soap film are considered from the hypothesis that equilibrium exists. As approximately no flow of material takes place in a direction perpendicular to the film, there is no objection against the assumption of equilibrium in this direction. This leads to the equation

$$(\delta\varphi/\delta r) + (tg\varphi/r) = (h\sigma/2T) \tag{1}$$

However, in the direction tangential to the film, gravitational force can cause flow of material. A superficial observation of a soap film shows that if the film makes an angle with the horizontal plane, the thickness at the upper edge rapidly decreases.

In the equation of motion of the liquid the surface tension forces cancel. If viscosity forces are not taken into account, only gravitational force remains.

$$dv/dt = (\delta v/\delta t) + v\cos\varphi(\delta v/\delta r) = g\sin\varphi \tag{2}$$

The continuity equation gives

$$\cos\varphi(\delta/\delta r)(rhv) = r(\delta h/\delta t) \tag{3}$$

Equations (1–3) are partial differential equations for the unknown functions h, v, φ. They are not solved easily. Moreover, in reality, viscosity forces are to be taken in because, at the edges, adhesion forces prevent the film itself from flowing. The flow in the inner parts of the film therefore will be larger than in the outer parts. In this case, Eqs. (2) and (3) read

$$\frac{\delta v}{\delta t} + v\cos\varphi\frac{\delta v}{\delta r} = \mu\frac{\delta^2 v}{dx^2} + g\sin\varphi \tag{2a}$$

$$\cos\varphi\frac{\delta}{\delta r}\left(r\int_{-h/2}^{h/2} v\,dx\right) = r\frac{\delta h}{\delta t} \tag{3a}$$

Because of the constancy of the surface tension not only $v = 0$ but also $(\delta v/\delta x) = 0$ will prevail at the surfaces $x = \pm h/2$. The velocities will be smaller therefore and it will take more time before the film breaks down if the solution is of high viscosity.

Reference

[1] Galletly, R. A., "A note on a soap-film paradox," J. Aerospace Sci. 29, 1487–1488 (1962).

Note on Solution of a System of Three-Moment Equations

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IN a recent paper, Wolff[1] gives a solution for a system of N-1 three-moment equations by expressing the inverted matrix of the system as a sum of N matrices, each N-1 by N-1 in size. By making use of recursion formulas given by Schechter,[2] it is possible to express the elements of the inverted matrix in terms of an invariant set of N numbers, a form that involves a small percentage of the calculation required in Ref. 1.

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For the particular case of the three-moment equation given in Ref. 1,

$$M_{n-1} + 4M_n + M_{n+1} = -G_n \qquad n = 1,2,3,\ldots,N-1 \tag{1}$$

$$M_0 = 0 \qquad M_N = 0$$

Schechter's[2] recursion formulas reduce to

$$u_1 = -G_1$$
$$u_n = -H_{n-1}G_n - u_{n-1} \tag{2}$$
$$n = 2,3,\ldots,N-1$$

$$H_0 = 1 \qquad H_1 = 4 \qquad H_2 = 15$$
$$H_n = 4H_{n-1} - H_{n-2} \tag{3}$$
$$n = 2,3,\ldots,N-1$$

$$M_{N-1} = u_{N-1}/H_{N-1}$$
$$M_{N-2} = -G_{N-1} - 4M_{N-1} \tag{4}$$
$$M_{n-1} = -G_n - 4M_n - M_{n+1}$$
$$n = N-2, N-3,\ldots,2$$

Because of the simple form of the recursion formula in Eq. (2), it can be replaced by a simple sum to give u_{N-1}, the only value needed to start the recursion for the moments in Eq. (4):

$$\frac{u_{N-1}}{H_{N-1}} = M_{N-1} = \frac{1}{H_{N-1}}\sum_{i=1}^{N-1}(-1)^{N-i}H_{i-1}G_i \tag{5}$$

From the matrix form of Eq. (1)

$$QM = -G \tag{6}$$

the solution for M is

$$M = -Q^{-1}G \tag{7}$$

Examination of Eqs. (4) and (5) shows the elements q_{nm} of Q^{-1} on and to the left of the main diagonal have the form

$$q_{nm} = (-1)^{2N-n-m}\left(\frac{H_{N-n-1}H_{m-1}}{H_{N-1}}\right) \tag{8}$$
$$m = 1,2,3,\ldots,n$$
$$n = 1,2,3,\ldots,N-1$$

Since Q^{-1} is symmetrical, the elements to the right of the main diagonal are obtained by interchanging n and m in Eq (8).

Thus the solution for the moments is given either by Eqs. (5) and (4) or by Eqs. (8) and (7). Only the fixed set of numbers 1,4,15,56,209,780, . . . , given in Eq. (3) and the values of G_n, which represent the applied loads and temperatures acting between supports $n-1$ and $n+1$, are needed to make the calculations by either of the two methods. The amount of calculation by either procedure appears to be much smaller than that required in Ref. 1.

References

[1] Wolff, T., "Direct solution of the three-moments equation," AIAA J. 1, 718 (1963).

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